

Midpoint and Distance Formulas

Main Ideas

- Find the midpoint of a segment on the coordinate plane.
- Find the distance between two points on the coordinate plane.

GET READY for the Lesson

A square grid is superimposed on a map of eastern Nebraska where emergency medical assistance by helicopter is available from both Lincoln and Omaha. You can use the formulas in this lesson to determine whether the site of an emergency is closer to Lincoln or to Omaha.



Each side = 10 miles

The Midpoint Formula Recall that point M is the midpoint of segment PQ if M is between P and Q and $PM = MQ$. There is a formula for the coordinates of the midpoint of a segment in terms of the coordinates of the endpoints.

Study Tip

Midpoints

The coordinates of the midpoint are the means of the coordinates of the endpoints.

KEY CONCEPT

Midpoint Formula

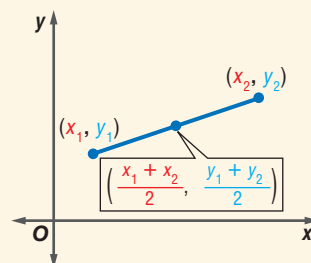
Words

If a line segment has endpoints at (x_1, y_1) and (x_2, y_2) , then the midpoint of the segment has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Symbols

midpoint = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Model

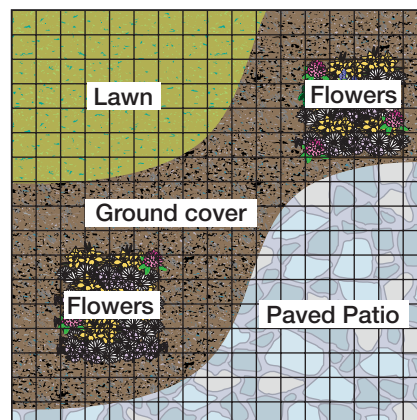


You will show that this formula is correct in Exercise 38.

EXAMPLE Find a Midpoint

- 1 LANDSCAPING** A landscape design includes two square flower beds and a sprinkler halfway between them. Find the coordinates of the sprinkler if the origin is at the lower left corner of the grid.

The centers of the flower beds are at $(4, 5)$ and $(14, 13)$. The sprinkler will be at the midpoint of the segment joining these points.



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + 14}{2}, \frac{5 + 13}{2}\right)$$

$$= \left(\frac{18}{2}, \frac{18}{2}\right) \text{ or } (9, 9)$$

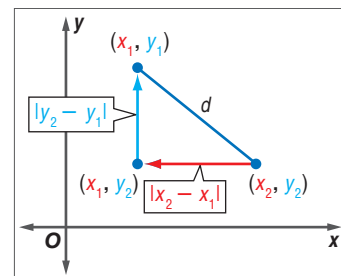
The sprinkler will have coordinates (9, 9).

CHECK Your Progress

1. The landscape architect decides to place a bench in the middle of the lawn area. Find the coordinates of the bench using the endpoints (0, 17) and (7, 11).

The Distance Formula Recall that the distance between two points on a number line whose coordinates are a and b is $|a - b|$ or $|b - a|$. You can use this fact and the Pythagorean Theorem to derive a formula for the distance between two points on a coordinate plane.

Suppose (x_1, y_1) and (x_2, y_2) name two points. Draw a right triangle with vertices at these points and the point (x_1, y_2) . The lengths of the legs of the right triangle are $|x_2 - x_1|$ and $|y_2 - y_1|$. Let d represent the distance between (x_1, y_1) and (x_2, y_2) .



$$c^2 = a^2 + b^2$$

Pythagorean Theorem

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Substitute.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$|x_2 - x_1|^2 = (x_2 - x_1)^2; |y_2 - y_1|^2 = (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the nonnegative square root of each side.

Study Tip

Distance

In mathematics, just as in life, distances are always nonnegative.

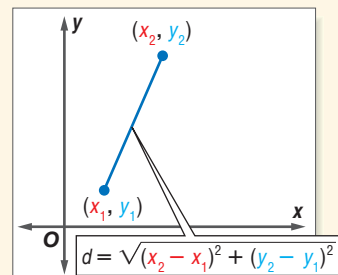
KEY CONCEPT

Distance Formula

Words The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Model



EXAMPLE Find the Distance Between Two Points

- 2 Find the distance between $A(-3, 6)$ and $B(4, -4)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance Formula

$$= \sqrt{[4 - (-3)]^2 + (-4 - 6)^2}$$

Let $(x_1, y_1) = (-3, 6)$ and $(x_2, y_2) = (4, -4)$.

$$= \sqrt{7^2 + (-10)^2}$$

Subtract.

$$= \sqrt{49 + 100} \text{ or } \sqrt{149} \text{ units}$$

CHECK Your Progress

2. Find the distance between $R(-6, 5)$ and $S(-3, -2)$.



STANDARDIZED TEST EXAMPLE

A coordinate grid is placed over a California map. Bakersfield is located at $(3, -7)$, and Fresno is located at $(-7, 9)$. If Tulare is halfway between Bakersfield and Fresno, which is the closest to the distance in coordinate units from Bakersfield to Tulare?

A 6.25

B 9.5

C 12.5

D 19

Read the Test Item

The question asks us to find the distance between one city and the midpoint. Find the midpoint and then use the Distance Formula.

Solve the Test Item

Use the Midpoint Formula to find the coordinates of Tulare.

$$\begin{aligned}\text{midpoint} &= \left(\frac{3 + (-7)}{2}, \frac{(-7) + 9}{2} \right) && \text{Midpoint Formula} \\ &= (-2, 1) && \text{Simplify.}\end{aligned}$$

Use the Distance Formula to find the distance between Bakersfield $(3, -7)$ and Tulare $(-2, 1)$.

$$\begin{aligned}\text{distance} &= \sqrt{(-2 - 3)^2 + (1 - (-7))^2} && \text{Distance Formula} \\ &= \sqrt{(-5)^2 + 8^2} && \text{Subtract.} \\ &= \sqrt{89} \text{ or about } 9.4 && \text{Simplify.}\end{aligned}$$

The answer is B.

Test-Taking Tip

In order to check your answer, find the distance between Tulare and Fresno. Since Tulare is at the midpoint, these distances should be equal.

CHECK Your Progress

3. The coordinates for points A and B are $(-4, -5)$ and $(10, -7)$, respectively. Find the distance between the midpoint of A and B and point B .

F $\sqrt{10}$ units

H $\sqrt{50}$ units

G $5\sqrt{10}$ units

J $10\sqrt{5}$ units

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CHECK Your Understanding

Example 1
(pp. 562–563)

Find the midpoint of the line segment with endpoints at the given coordinates.

1. $(-5, 6), (1, 7)$

2. $(8, 9), (-3, -4.5)$

3. $(13, -4), (10, 14.6)$

4. $(-12, -2), (-3.5, -7)$

Example 2
(p. 563)

Find the distance between each pair of points with the given coordinates.

5. $(2, -4), (10, -10)$

6. $(7, 8), (-4, 9)$

7. $(0.5, 1.4), (1.1, 2.9)$

8. $(-4.3, 2.6), (6.5, -3.4)$

Example 3
(p. 564)

9. **STANDARDIZED TEST PRACTICE** The map of a mall is overlaid with a numeric grid. The kiosk for the cell phone store is halfway between Terry's Ice Cream and the See Clearly eyeglass store. If the ice cream store is at $(2, 4)$ and the eyeglass store is at $(78, 46)$, find the distance the kiosk is from the eyeglass store.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
10–15	1
16–21	2, 3

Find the midpoint of the line segment with endpoints at the given coordinates.

10. $(8, 3), (16, 7)$

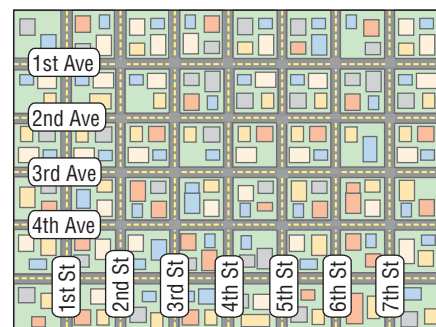
11. $(-5, 3), (-3, -7)$

12. $(6, -5), (-2, -7)$

13. $(5, 9), (12, 18)$

14. **GEOMETRY** Triangle MNP has vertices $M(3, 5)$, $N(-2, 8)$, and $P(7, -4)$. Find the coordinates of the midpoint of each side.

15. **REAL ESTATE** In John's town, the numbered streets and avenues form a grid. He belongs to a gym at the corner of 12th Street and 15th Avenue, and the deli where he works is at the corner of 4th Street and 5th Avenue. He wants to rent an apartment halfway between the two. In what area should he look?



Find the distance between each pair of points with the given coordinates.

16. $(-4, 9), (1, -3)$

17. $(1, -14), (-6, 10)$

18. $(-4, -10), (-3, -11)$

19. $(9, -2), (12, -14)$

20. $(0.23, 0.4), (0.68, -0.2)$

21. $(2.3, -1.2), (-4.5, 3.7)$

22. **GEOMETRY** Quadrilateral $RSTV$ has vertices $R(-4, 6)$, $S(4, 5)$, $T(6, 3)$, and $V(5, -8)$. Find the perimeter of the quadrilateral.

23. **GEOMETRY** Triangle BCD has vertices $B(4, 9)$, $C(8, -9)$, and $D(-6, 5)$. Find the length of median $\overline{BP}$. (Hint: A median connects a vertex of a triangle to the midpoint of the opposite side.)

Find the midpoint of the line segment with endpoints at the given coordinates. Then find the distance between the points.

24. $(-3, -\frac{2}{11}), (5, \frac{9}{11})$

25. $(0, \frac{1}{5}), (\frac{3}{5}, -\frac{3}{5})$

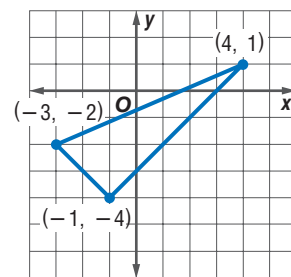
26. $(2\sqrt{3}, -5), (-3\sqrt{3}, 9)$

27. $(\frac{2\sqrt{3}}{3}, \frac{\sqrt{5}}{4}), (-\frac{2\sqrt{3}}{3}, \frac{\sqrt{5}}{2})$

28. **GEOMETRY** Find the perimeter and area of the triangle at the right.

29. **GEOMETRY** A circle has a radius with endpoints at $(2, 5)$ and $(-1, -4)$. Find the circumference and area of the circle.

30. **GEOMETRY** Circle Q has a diameter $\overline{AB}$. If A is at $(-3, -5)$ and the center of the circle is at $(2, 3)$, find the coordinates of B .



GEOGRAPHY For Exercises 31 and 32, use the following information.

The U.S. Geological Survey (USGS) has determined the official center of the United States.

31. Approximate the center of the United States. Describe your method.

32. **RESEARCH** Use the Internet or other reference to look up the USGS geographical center of the United States. How does the location given by USGS compare to the result of your method?

EXTRA PRACTICE
See pages 911, 935.
Math online
Self-Check Quiz at
algebra2.com

H.O.T. Problems

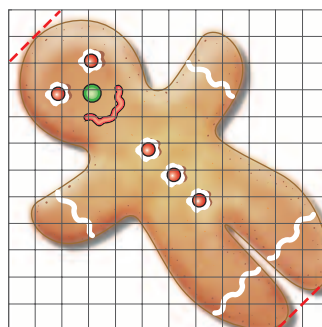
TRAVEL For Exercises 33 and 34, use the figure at the right, where a grid is superimposed on a map of a portion of the state of Alabama.



33. How far is it from Birmingham to Montgomery if each unit on the grid represents 40 miles?

34. How long would it take a plane to fly from Huntsville to Montgomery if its average speed is 180 miles per hour?

35. WOODWORKING A stage crew is making the set for a children's play. They want to make some gingerbread shapes out of leftover squares of wood with sides measuring 1 foot. They can make taller shapes by cutting them out of the wood diagonally. To the nearest inch, how tall is the gingerbread shape in the drawing?



36. OPEN ENDED Find two points that are $\sqrt{29}$ units apart.

37. REASONING Identify all of the points that are equidistant from the endpoints of a given segment.

38. CHALLENGE Verify the Midpoint Formula. (*Hint: You must show that the formula gives the coordinates of a point on the line through the given endpoints and that the point is equidistant from the endpoints.*)

39. Writing in Math Explain how to use the Distance Formula to approximate the distance between two cities on a map.

STANDARDIZED TEST PRACTICE

40. ACT/SAT Point $D(5, -1)$ is the midpoint of segment $\overline{CE}$. If point C has coordinates $(3, 2)$, what are the coordinates of point E ?

- A $(8, 1)$
- B $(7, -4)$
- C $(2, -3)$
- D $(4, \frac{1}{2})$

41. REVIEW If $\log_{10} x = -3$, what is the value of x ?

- F $x = 1000$
- G $x = \frac{1}{1000}$
- H $x = \sqrt{\frac{1}{100}}$
- J $x = -\sqrt{\frac{1}{100}}$

Spiral Review

42. COMPUTERS Suppose a computer that costs \$3000 new is only worth \$600 after 3 years. What is the average annual rate of depreciation? (Lesson 9-6)

Solve each equation. Round to the nearest ten-thousandth. (Lesson 9-5)

43. $3e^x - 2 = 0$

44. $e^{3x} = 4$

45. $\ln(x + 2) = 5$

GET READY for the Next Lesson

PREREQUISITE SKILL Write in the form $y = a(x - h)^2 + k$. (Lesson 5-5)

46. $y = x^2 + 6x + 9$

47. $y = 2x^2 + 20x + 50$

48. $y = -3x^2 - 18x - 10$